Temporal Tomographic Imaging by Greedy Fekete and Golden Means Sampling

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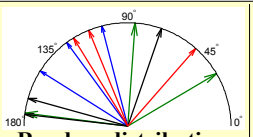
ABSTRACT

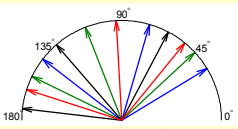
Electron Paramagnetic Resonance Imaging (EPRI) can provide insight into in vivo anatomic and functional imaging of free radicals and paramagnetic molecules and their role in disease in small animal models. There are many static sampling strategies available for tomography to determine which gradient angles are acquired: equal linear angle, equal solid angle, Fekete points, and Monte-Carlo-based sequences [1]. To date, no sampling strategy has addressed the temporal aspect of EPRI. All of the aforementioned EPRI sampling strategies assume a static object; repeating the same angles over time provides highly redundant and correlated information. Our goal was to develop a more robust sampling strategy and improve the application of EPRI to dynamic imaging.

INTRODUCTION

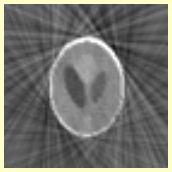
Consider the simplified case of distributing angles in 2D. While equal linear angle sampling in 2D provides optimal spatial resolution, it does not specify the order of the angles with respect to time. Previous work showed the superiority of using the Fibonacci sequence to determine both the angles and their order [2].



Random distribution



Fibonacci



The different temporal groups of arrows above (↑, ↑, ↑, ↑) are nearly uniform both spatially and temporally in the Fibonacci sequence, unlike in the purely random distribution. Reconstructed images likewise have lower noise and better contrast. The Fibonacci sequence can be generalized to 3D temporal sampling by solving an eigenvector problem, resulting in the Golden Means distribution [2].

FEKETE POINTS

Another approach to generating angles is to minimizing the following function, which we refer to as Fekete points [3]:

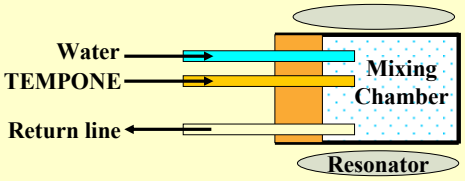
$$U = \tfrac{1}{2} \Sigma \, | v_i - v_j |^{-t} \qquad 1 \le i < j \le N \qquad \text{Eq(1)}$$

Eq(1) is Coulumb's law when t=1, where U is the potential energy of N point charges on the sphere. Each point v has the coordinates with azimuth angle α and polar angle β as follows:

$$v_x = \cos(\beta)\cos(\alpha); \; v_y = \cos(\beta)\sin(\alpha); \; v_z = \sin(\beta)$$

METHODS

We compared two different temporal distributions in 3D EPR imaging of a dynamic phantom. A 6 ml chamber was filled with 10 mM 4-oxo-2,2,6,6- tetra- methyl- piperidine- l-oxyl (TEMPONE) diluted by deionized water at a constant rate of 5 ml/min while EPR projections were continuously acquired.



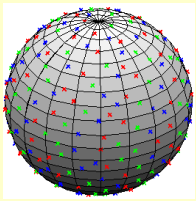
Distribution 1: Golden Means

Based on the Fibonacci sequence generalized to 3D, Golden Means generates angles as follows:

$$\alpha(m) = 2\pi \, \mathrm{frac}(m \, \Phi_1) \qquad \beta(m) = \cos^{-1}(\, \mathrm{frac}(m \, \Phi_2) \,) \qquad m = 1,2\ldots,N$$

where the azimuth angle is α and the polar angle is β, frac() returns the fractional component of a real number, N projections are required, and Φ_1 and Φ_2 are the eigenvector components of the solution to the generalized 3D Fibonacci problem ($\Phi_1 \approx 0.4656$, $\Phi_2 \approx 0.6823$).

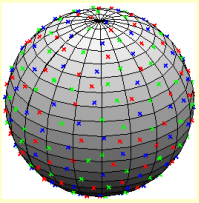
Three groups of 50 projections (x, x, x) generated by Golden Means



Distribution 2: Greedy Fekete

Equation (1) was iteratively solved for t=1 and N=2000 points using a random initialization and gradient descent minimization of U. To add the dynamic component to the distribution, the projections were re-ordered such that adding each point caused the minimum increase in U.

Three groups of 50 projections (x, x, x) generated by Greedy Fekete

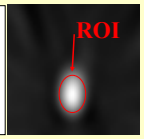


EPRI System

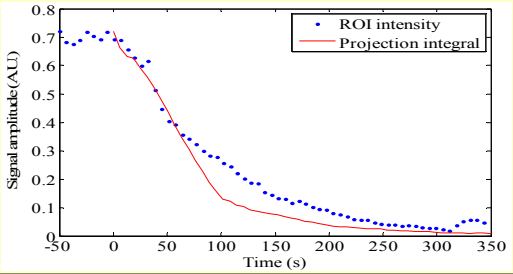
A 1.2 GHz continuous wave EPRI system was used to acquire projections for both distributions with the following acquisition parameters: 0.1 s per projection, 2 mT sweep field width, 0.04 mT center field, 0.07 mT modulation field amplitude, 100 kHz modulation frequency.

RESULTS

3D image volumes were reconstructed from groups of 64 projections using filtered backprojection after hyperfine correction and deconvolution.

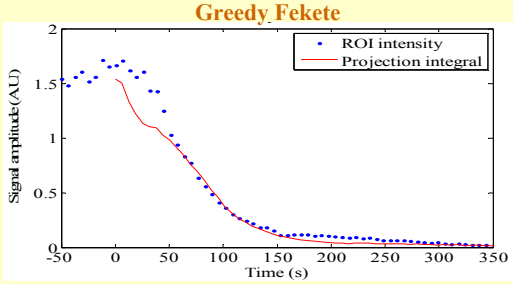


Golden Means



The mean signal intensity of a 3D region of interest (ROI) was plotted against each reconstructed timepoint. The integral of each projection was assumed to be the ground truth because a drop in signal amplitude was the result of dilution.

Greedy Fekete



The Greedy Fekete distribution (lower graph) appears to capture the temporal dilution better than Golden Means because the ROI intensities more closely match the integral of each projection.

DISCUSSION AND CONCLUSION

Greedy Fekete (GF) appears to capture the dynamic changes better than Golden Means (GM) in the dilution phantom. GM maintains an approximate uniformity for any set of projections grouped together in time, but uniformity of larger sets is suboptimal. This can be seen by visually inspecting the distributions on the unit sphere at left.

In contrast, GF for the first *i* projections always is as uniform as possible for any *i*, which appears to be an advantage for this type of experiment. In vivo applications of GF sampling are in progress, including EPR measurements of myocardial oxygenation, redox state, pH, and nitric oxide metabolism.

REFERENCES AND ACKNOWLEDGEMENTS

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[2] Temporal Stability of Adaptive 3D Radial MRI Using Multidimensional Golden Means. Chan et al. MRM 2009; 61(2):354–363.
[3] Nested Uniform Sampling For Multiresolution 3-d Tomography. Ahmad et al. IEEE Conf (ICASSP) 14-19 March 2010 (Dallas, TX): 461-4.

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